Find the Mistake

All of the following problems contain a mistake. Identify and correct each one.

1. **Section 2.2**: The negation of "$1 < a < 5$" is "$1 \geq a \geq 5$."
2. **Section 2.2**: "P only if Q" means "if Q then P."
3. **Section 3.2**
   (a) The negation of "For all real numbers x, if $x > 2$ then $x^2 > 4$" is "For all real numbers x, if $x > 2$ then $x^2 \leq 4$."
   (b) The negation of "For all real numbers x, if $x > 2$ then $x^2 > 4$" is "There exist real numbers x such that if $x > 2$ then $x^2 \leq 4$."
   (c) The negation of "For all real numbers x, if $x > 2$ then $x^2 > 4$" is "There exists a real number x such that $x > 2$ and $x^2 < 4$."
4. **Section 3.2**: The contrapositive of "For all real numbers x, if $x > 2$ then $x^2 > 4$" is "For all real numbers x, if $x \leq 2$ then $x^2 \leq 4$."
5. **Section 3.3**: Statement: $\exists$ a real number x such that $\forall$ real numbers y, $x + y = 0$. Proposed negation: $\forall$ real numbers x, if y is a real number then $x + y \neq 0$.
6. **Section 4.1**: A person is asked to prove that the square of any odd integer is odd. Toward the end of a proof the person writes: "Therefore $n^2 = 2k + 1$, which is the definition of odd."
7. **Section 4.1**: *Prove:* The square of any even integer is even.

   Beginning of proof: Suppose that r is any integer. Then if m is any even integer, $m = 2r$. . . .
8. **Section 4.1**: *Prove directly from the definition of even:* For all even integers n, $(-1)^n = 1$.

   Beginning of proof: Suppose n is any even integer. Then $n = 2r$ for some integer r. By substitution, $(-1)^n = (-1)^{2r} = 1$ because $2r$ is even....
9. **Section 4.1**: *Prove directly from the definition of even:* For all even integers n, $(-1)^n = 1$.

   Beginning of proof: Suppose n is any even integer. Then $n = 2r$ for some integer r. By substitution, $(-1)^{2r} = ((-1)^2)^r$....
10. **Section 4.3**: *Prove:* For all integers a and b, if a and b are divisible by 3 then $a+b$ is divisible by 3.

    Beginning of proof: Suppose that for all integers a and b, if a and b are divisible by 3 then $a + b$ is divisible by 3. By definition of divisibility, ...
11. **Section 4.3**: *Prove:* For all integers a, if 3 divides a, then 3 divides a^2.

    Beginning of proof: Suppose a is any integer such that 3 divides a. Then $a = 3k$ for any integer k....
12. **Section 4.3**: *Prove:* For all integers a, if $a = 3b+1$ for some integer b, then $a^2 - 1$ is divisible by 3.

    Beginning of proof: Let a be any integer such that $a = 3b + 1$ for some integer b. We will prove that $a^2 - 1$ is divisible by 3. This means that $a^2 - 1 = 3q$ for some integer q. Then $(3b + 1)^2 - 1 = 3q$, and, since q is an integer, by definition of divisibility, $a^2 - 1$ is divisible by 3....

13. **Section 4.4**: *Prove:* For all integers a, $a^2 - 2$ is not divisible by 3.

    Beginning of proof: Suppose a is any integer. By the quotient-remainder theorem with divisor $d = 3$, there exist unique integers q and r such that $a = 3q + r$, where $0 < r \leq 3$....

14. **Section 4.6**: *Prove by contradiction:* The product of any irrational number and any rational number is irrational.

    Beginning of proof: Suppose not. That is, suppose the product of any irrational number and any rational number is rational....

15. **Section 4.6**: The negation of "n is not divisible by any prime number greater than 1 and less than or equal to $\sqrt{n}$" is "n is divisible by any prime number greater than 1 and less than or equal to $\sqrt{n}$."

16. **Section 5.2**: The equation $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$ is true for $n = 1$ because $1 + 2 + 3 + \cdots + 1 = \dfrac{1(1+1)}{2}$ is true.

17. **Section 5.2**: The equation $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$ is true for $n = 1$ because

$$1 = \frac{1(1+1)}{2} \Rightarrow 1 = \frac{2}{2} \Rightarrow 1 = 1.$$

18. **Section 5.2**: *Prove by mathematical induction:* For all integers $n \geq 1$,

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}.$$

    Beginning of proof: Let the property $P(n)$ be

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \text{ for all integers } n \geq 1....$$

19. **Section 6.1**: Given sets A and B, to show that A is a subset of B, we must show that there is an element x such that x is in A and x is in B.

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21. **Section 7.2**: To prove that $F\colon A \to B$ is one-to-one, assume that if $F(x_1) = F(x_2)$ then $x_1 = x_2$.

22. **Section 7.2**: To prove that $F\colon A \to B$ is one-to-one, we must show that for all x_1 and x_2 in A, $F(x_1) = F(x_2)$ and $x_1 = x_2$.

23. **Section 8.2**: Define a relation R on the set of all integers by $a\,R\,b$ if, and only if, $ab > 0$. To show that R is symmetric, assume that for all integers a and b, $a\,R\,b$. We will show that $b\,R\,a$.