

## Review Guide: Chapter 8

**Definitions:** How are the following terms defined?

- congruence modulo 2 relation (p. 443)
- inverse of a relation from a set  $A$  to a set  $B$  (p. 444)
- relation on a set (p. 446)
- directed graph of a relation on a set (p. 446)
- $n$ -ary relation (and binary, ternary, quaternary relations) (p. 447)
- reflexive, symmetric, and transitive properties of a relation on a set (p. 450)
- congruence modulo 3 relation (p. 455)
- transitive closure of a relation on a set (p. 457)
- equivalence relation on a set (p. 462)
- equivalence class (p. 465)
- congruence modulo  $n$  relation (p. 471)
- representative of an equivalence class (p. 472)
- $m$  is congruent to  $n$  modulo  $d$  (p. 473)
- plaintext and cyphertext (p. 478)
- residue of  $a$  modulo  $n$  (p. 481)
- $d$  is a linear combination of  $a$  and  $b$  (p. 486)
- $a$  and  $b$  are relatively prime;  $a_1, a_2, \dots, a_n$  are pairwise relatively prime (p. 488)
- an inverse of  $a$  modulo  $n$  (p. 489)
- antisymmetric relation (p. 499)
- partial order relation (p. 500)
- lexicographic order (p. 502)
- Hasse diagram (p. 503)
- $a$  and  $b$  are comparable (p. 505)
- poset (p. 506)
- total order relation (p. 506)
- chain, length of a chain (p. 506)
- maximal element, greatest element, minimal element, least element (p. 507)
- topological sorting (p. 507)
- compatible partial order relations (p. 508)
- PERT and CPM (p. 510)
- critical path (p. 512)

### Properties of Relations on Sets and Equivalence Relations

- How do you show that a relation on a finite set is reflexive? symmetric? transitive? (pp. 450-452)
- How do you show that a relation on an infinite set is reflexive? symmetric? transitive? (pp. 453-456)
- How do you show that a relation on a set is not reflexive? not symmetric? not transitive? (pp. 451-454)
- How do you find the transitive closure of a relation? (p. 457)
- What is the relation induced by a partition of a set? (p. 460)
- Given an equivalence relation on a set  $A$ , what is the relationship between the distinct equivalence classes of the relation and the set  $A$ ? (p. 469)
- In what way are rational numbers equivalence classes? (pp. 473-474)

**Cryptography**

- How does the Caesar cipher work? (*p. 478*)
- If  $a$ ,  $b$ , and  $n$  are integers with  $n > 1$ , what are some different ways of expressing the fact that  $n \mid (a - b)$ ? (*p. 480*)
- If  $n$  is an integer with  $n > 1$ , is congruence modulo  $n$  an equivalence relation on the set of all integers? (*p. 481*)
- How do you add, subtract, and multiply integers modulo an integer  $n > 1$ ? (*p. 482*)
- What is an efficient way to compute  $a^k$  where  $a$  is an integer with  $a > 1$  and  $k$  is a large integer? (*pp. 484-485*)
- How do you express the greatest common divisor of two integers as a linear combination of the integers? (*p. 487*)
- When can you find an inverse modulo  $n$  for a positive integer  $a$ , and how do you find it? (*pp. 488-489*)
- How do you encrypt and decrypt messages using RSA cryptography? (*pp. 491-492*)
- What is Euclid's lemma? How is it proved? (*p. 492*)
- What is Fermat's little theorem? How is it proved? (*p. 494*)
- Why does the RSA cipher work? (*pp. 494-496*)

**Partial Order Relations**

- How do you show that a relation on a set is or is not antisymmetric? (*pp. 499-500*)
- If  $A$  is a set with a partial order relation  $R$ ,  $S$  is a set of strings over  $A$ , and  $a$  and  $b$  are in  $S$ , how do you show that  $a \preceq b$ , where  $\preceq$  denotes the lexicographic ordering of  $S$ ? (*p. 502*)
- How do you construct the Hasse diagram for a partial order relation? (*p. 503*)
- How do you find a chain in a partially ordered set? (*p. 506*)
- Given a set with a partial order, how do you construct a topological sorting for the elements of the set? (*p. 508*)
- Given a job scheduling problem consisting of a number of tasks, some of which must be completed before others can be begun, how can you use a partial order relation to determine the minimum time needed to complete the job? (*pp. 511-512*)