Assignment 8 (Due date: 4/30/2010, Friday, in class)

Your name:	Grade:

## Write your solutions below/beside the question texts.

1. Using big O() (asymptotic) notations to describe the following algorithms (e.g., O(1),  $O(n^2)$ ...):

```
a. sum = 0;
  for (i=0; i<3; i++)
      for (j=0; j<n; j++)
           sum++;
b. sum = 0;
  for (i=0; i<8888; i++)
      for (j=0; j<9999; j++)
           sum++;
c. sum = 0;
  for (i = 0; i < n; i++) {
       if (is even(i)) {
           for (j = 0; j < n; j++)
           sum++;
       } else
           sum = sum + n;
  }
d. for (i=0; i<n; i++) {
      for (j=0; j<n; j++)</pre>
           A[i] = random(n); // assume random() is O(1)
       sort(A, n); // assume sort() is O(n log n)
  }
```

2. Provide a formal definition of big O notation.

3. Identify some functions that can be represented completely in tabular form (12.1.1).

4. Identify some functions whose outputs can be described as an algebraic expression involving their inputs (12.1.2).

5. Read the article "*In an Ancient Game, Computing's Future*" (the handout given in the class) and briefly describe why the ancient game Go provides a very good example for the computability and complexity? (You may use some examples in the article for clarify your arguments).

6. Identify a function that cannot be described in terms of an algebraic formula. Is your function nonetheless computable (12.1.3)?

7. Describe a Turing machine that replaces a string of 0s and 1s with a single 0 (12.2.2).

8. Describe a Turing machine that ultimately halts for some inputs but never halts for others (12.2.5).

9. Briefly describe the Church-Turing thesis.

10. Describe the function computed by the following Bare Bones program, assuming the function's input is represented by x and its output by z (12.3.6).

clear z; while x not 0 do; incr z; incr z; decr x; end;

11. What is wrong with the following scenarios (12.4.3)?

In a certain community, everyone owns his or her own house. The house painter of the community claims to paint all those and only those houses that are not painted by their owners. 12. Suppose a problem can be solved by an algorithm in  $O(n^2)$  as well as another algorithm in  $O(2^n)$ . Will one algorithm always outperform the other (12.5.2)?

13. List all of the subcommittees that can be formed from a committee consisting of the two members Alice and Bill. List all the subcommittees that can be formed from the committee consisting of Alice, Bill, and Carol. What about the subcommittees from Alice, Bill, Carol, and David (12.5.3)?

14. Give an example of a polynomial problem. Give an example of a nonpolynomial problem. Give an example of an NP problem that as yet has not been shown to be a polynomial problem (12.5.4).

15. If the complexity of algorithm X is greater than that of algorithm Y, is algorithm X necessarily harder to understand than algorithm Y? Explain your answer (12.5.5).

16. Write a Bare Bones program that calculates the sum of the integers from 0 to x.

17. Design a Turing machine that places 0s in all the cells to the left of the current cell until it reaches a cell containing an asterisk.

18. Suppose we were faced with solving the Traveling Salesman problem in a context involving 15 cities in which any two cities were connected by a unique road. How many different paths through the cities would there be? How long would it take to compute the length of all of these paths assuming that the length of a path can be computed in one microsecond?

- 19. Which of the following problems are in the class P?
  - **a.** A problem with complexity  $n^2$
  - **b.** A problem with complexity  $n^2+2n$
  - **c.** A problem with complexity n!
- 20. Give one or two examples to illustrate the problems being deterministic and those being nondeterministic. And briefly describe NP problems in your own words (section 12.5).

21. Provide a formal/mathematical description of an Automaton by using 5-tuple.

22. Given the following DFA, transcribe it by using the formal automaton 5-tuple model.



23 (**Bonus**). Suppose we have obtained the following likelihood ratios (LSs) from data on P(E|D) and  $P(E|\sim D)$  (check the course website about AI' Expert System slides/links).

	Measles LS	Mumps LS
Spots	18	12
No spots	0.4	0.6
High Temp.	6	8
No Temp.	0.85	0.75

Suppose we also know that the prior odds of a child having measles are 0.08 and of mumps are 0.06. Now calculate the posterior odds for the following situations:

- 1. If John has spots but not temperature, calculate the posterior odds of measles.
- 2. If Sam has spots and high temperature, calculate the posterior odds of mumps.
- 3. If Jack has no spots but high temperature, calculate the posterior odds of measles.
- 4. If Mike has no spots and no temperature, calculate the posterior odds of mumps.