

Assignment 6  
(Due date: 12/2/2008/Tuesday, in class)

Your name:	Date:
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1. Using big  $O()$  (asymptotic) notations to describe the following algorithms (e.g.,  $O(1)$ ,  $O(n^2)$ ...):

**a.**

```
sum = 0;
for (i=0; i<3; i++)
    for (j=0; j<n; j++)
        sum++;
```

**b.**

```
sum = 0;
for (i=0; i<8888; i++)
    for (j=0; j<9999; j++)
        sum++;
```

**c.**

```
sum = 0;
for (i = 0; i < n; i++) {
    if (is_even(i)) {
        for (j = 0; j < n; j++)
            sum++;
    } else
        sum = sum + n;
}
```

**d.**

```
for (i=0; i<n; i++) {
    for (j=0; j<n; j++)
        A[i] = random(n); // assume random() is O(1)
    sort(A, n); // assume sort() is O(n log n)
}
```

2. Provide a formal definition of big O notation.

3. Identify some functions that can be represented completely in tabular form (12.1.1).

4. Identify some functions whose outputs can be described as an algebraic expression involving their inputs (12.1.2).



7. Describe a Turing machine that replaces a string of 0s and 1s with a single 0 (12.2.2).

8. Describe a Turing machine that ultimately halts for some inputs but never halts for others (12.2.5).

9. Briefly describe the Church-Turing thesis.

10. Describe the function computed by the following Bare Bones program (12.3.6).

```
clear z;  
while x not 0 do;  
    incr z;  
    incr z;  
    decr x;  
end;
```

11. What is wrong with the following scenarios (12.4.3)?

*In a certain community, everyone owns his or her own house. The house painter of the community claims to paint all those and only those houses that are not painted by their owners.*

12. Suppose a problem can be solved by an algorithm in  $O(n^2)$  as well as another algorithm in  $O(2^n)$ . Will one algorithm always outperform the other (12.5.2)?
13. List all of the subcommittees that can be formed from a committee consisting of the two members Alice and Bill. List all the subcommittees that can be formed from the committee consisting of Alice, Bill, and Carol. What about the subcommittees from Alice, Bill, Carol, and David (12.5.3)?

14. Give an example of a polynomial problem. Give an example of a nonpolynomial problem. Give an example of an NP problem that as yet has not been shown to be a polynomial problem (12.5.3).

15. If the complexity of algorithm X is greater than that of algorithm Y, is algorithm X necessarily harder to understand than algorithm Y? Explain your answer (12.5.5).

16. Write a Bare Bones program that calculates the sum of the integers from 0 to  $x$ .

17. Design a Turing machine that places 0s in all the cells to the left of the current cell until it reaches a cell containing an asterisk.



18. Suppose we were faced with solving the Traveling Salesman problem in a context involving 15 cities in which any two cities were connected by a unique road. How many different paths through the cities would there be? How long would it take to compute the length of all of these paths assuming that the length of a path can be computed in one microsecond?
19. Which of the following problems are in the class P?
- a. A problem with complexity  $n^2$
  - b. A problem with complexity  $n^2+2n$
  - c. A problem with complexity  $n!$
20. Give one or two examples to illustrate the problems being deterministic and those being nondeterministic. And briefly describe NP problems in your own words (section 12.5).

21. Provide a formal/mathematical description of an Automaton by using 5-tuple.

22. Given the following DFA, transcribe it by using the formal automaton 5-tuple model.

